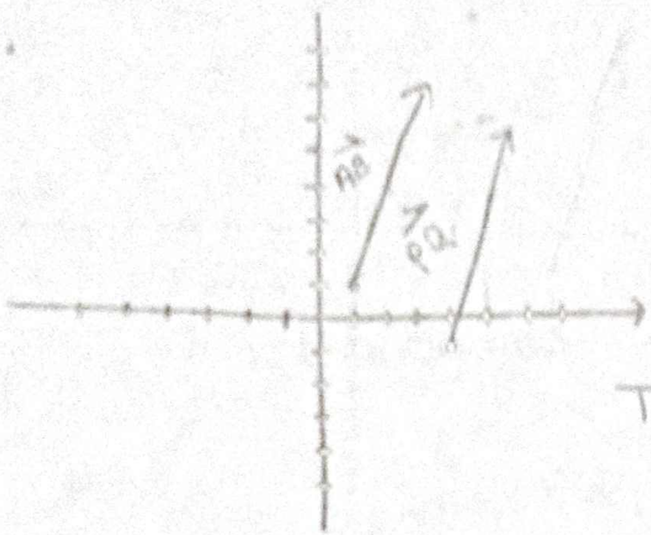
Math 1920 — Section 13.1 Additional Exercises

Solutions

1\.



the components of $\vec{AB}$ are: $\langle 2, 6 \rangle$.

the components of $\vec{PQ}$ are: $\langle 2, 6 \rangle$.

They are equivalent.

2\. Since $\mathbf{i} = \langle 1,0 \rangle$ and $\mathbf{j} = \langle 0,1 \rangle$, we have

$\mathbf{v} = \mathbf{i} - \mathbf{j} = \langle 1, -1 \rangle$.

So, the unit vector pointing in the direction of $\mathbf{v}$

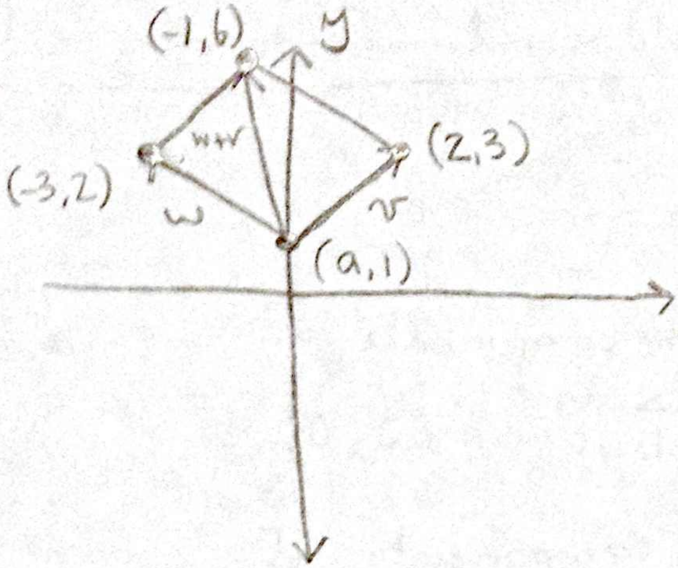
is: $e_v = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{1^2 + (-1)^2}}\langle 1, -1 \rangle = \frac{1}{\sqrt{2}}\langle 1, -1 \rangle$.

So, the vector of length 2 pointing in the direction of

$\mathbf{v}$ is: $2e_v = \frac{2}{\sqrt{2}}\langle 1, -1 \rangle = \langle -\sqrt{2}, -\sqrt{2} \rangle$.

Finally, the vector of length 2 pointing in the opposite direction of $\mathbf{v}$ is: $-2e_v = \boxed{\langle -\sqrt{2}, \sqrt{2} \rangle}$.

3.



The vectors w and v have components:

$$w = \langle -3-a, 1 \rangle \qquad v = \langle 2-a, 2 \rangle .$$

So, $w+v = \langle -3-a+2-a, 1+2 \rangle$

$$= \langle -1-2a, 3 \rangle = \langle -1-a, b-1 \rangle .$$

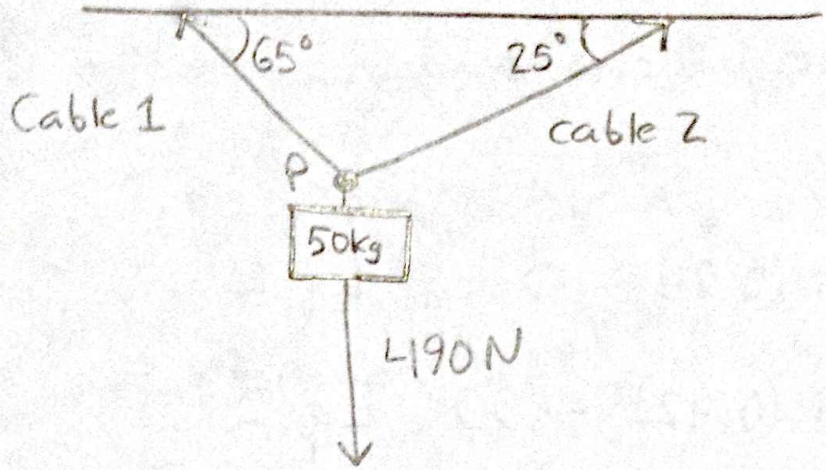
We get two equations:

$$\begin{cases} -1-2a = -1-a \\ 3 = b-1 \end{cases} \iff \begin{cases} a = 0 \\ b = 4 \end{cases} .$$

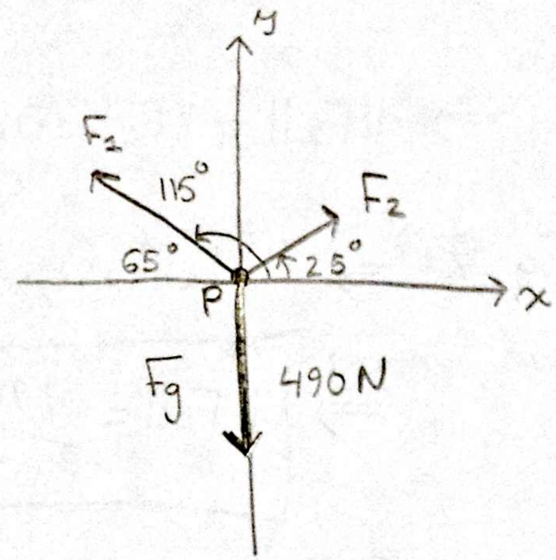
Note that we used the <u>Parallelogram Law</u>:

v + w is the vector pointing from the basepoint to the opposite vertex of the parallelogram formed by v and w.

4.



Force diagram:



Three forces act on the point P. The force due to gravity $50g = 490N$ ($g = 9.8 m/s^2$) acting vertically down, and two unknown forces F_1 and F_2.

Since P is not in motion, the net force is zero:

$F_1 + F_2 + F_g = 0$. We have: $F_1 = \|F_1\| \langle \cos 115°, \sin 115° \rangle$

$F_2 = \|F_2\| \langle \cos 25°, \sin 25° \rangle$

$F_g = \langle 0, -490 \rangle$.

So, $\|F_1\| \langle -0.42, 0.91 \rangle + \|F_2\| \langle 0.91, 0.42 \rangle + \langle 0, -490 \rangle = \langle 0, 0 \rangle$.

This gives us two equations and two unknowns:

$$-\|F_1\|(0.42) + \|F_2\|(0.91) = 0 \qquad \text{Eq. 1}$$

$$\|F_1\|(0.91) + \|F_2\|(0.42) = -490 \qquad \text{Eq. 2}$$

Solving: Eq. 1 $\Rightarrow \|F_2\| = \|F_1\|(0.46)$

Plugging into Eq. 2 $\Rightarrow \|F_1\|(0.91) + \|F_1\|(.1932) = 490$

$$\Rightarrow \boxed{\|F_1\| = \frac{490}{1.1032}\ N = 444\ N}$$

and $\boxed{\|F_2\| = 204\ N}$

Section 13.2 Additional Exercises

Solutions

. $r(t) = \langle 1, 2, -8\rangle + t\langle 2, 1, 3\rangle$

$$= \langle 1+2t, 2+t, -8-3t\rangle.$$

2. A suitable direction vector is $\mathbf{i} = \langle 1,0,0 \rangle$

since it is perpendicular to the yz-plane.

Then, a parametrization is $r(t) = \langle 4,9,8 \rangle + t\langle 1,0,0 \rangle$

$$= \langle 4+t, 9, 8 \rangle.$$

3. It is enough to show that $r_1(t)$ and $r_2(t)$ pass through a common point and that they are parallel.

Both pass through $P = (3,-1,4)$

since $r_1(0) = (3,-1,4)$ and $r_2(2) = (3,-1,4)$.

To show that the lines are parallel it is enough to show that their direction vectors are parallel. Since

$\langle 8,12,-6 \rangle = 2\langle 4,6,-3 \rangle$ we are done.

4. We need to find t_1 and t_2 such that

$r_1(t_1) = r_2(t_2)$.

This gives us three equations:

$$\begin{cases} t_1 = 2 + t_2 & (1) \\ 1 + t_1 = 4t_2 & (2) \\ 1 + 2t_1 = 3 + 4t_2 & (3) \end{cases}$$

Solving: plugging in Eq. 1 into Eq (2) gives:

$$3 + t_2 = 4t_2 \Rightarrow t_2 = 1$$

and $t_1 = 3$.

Checking, we get:

$r_1(3) = \langle 3, 4, 7 \rangle$

$r_2(1) = \langle 3, 4, 7 \rangle$.

$\boxed{\text{So, they intersect at the point } (3, 4, 7).}$